Global reconstruction in application to multichannel communication

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We propose a method for restoration of time-dependent control parameters of dynamical system using the technique of global reconstruction. The technique presented is applied to multichannel confidential communication by means of parameter modulation. [S1063-651X(98)11001-2]

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One of the methods for mathematical modeling of experimental data is the global reconstruction of dynamical systems (DS) [1,2]. The algorithm for global reconstruction has two stages. At the first stage the embedding dimension N is chosen and the phase portrait of DS of one-dimensional realization is restored [3]. At the second stage the general form of the mathematical model

$$\frac{d\vec{x}}{dt}=\vec{F}(\vec{x},\vec{\mu}^0),\quad \vec{x}\in\mathbb{R}^N,\quad \vec{\mu}^0\in\mathbb{R}^M, \tag{1}$$

is stated [the form of nonlinear vector function $\vec{F}(\vec{x},\vec{\mu}^0)$ is indicated *a priori*] and evolution equations are concretized, i.e., the value of parameter vector $\vec{\mu}^0$ is found.

In practice we deal with scalar time series produced by unknown DSs, so that it is impossible to indicate even approximately the form of right-hand parts of Eq. (1). Usually nonlinear functions on the right-hand side of Eq. (1) are defined as a polynomial expansion and the expansion coefficients are fitted numerically.

The task of global reconstruction on a scalar time series is related to the class of ill-posed inverse problems because there is no possibility to indicate uniquely the *a priori* form of nonlinearities $F_j(\vec{x},\vec{\mu}^0)$. Therefore, an arbitrary choice of the right-hand side of Eq. (1) is a serious shortcoming of the method of global reconstruction.

The situation changes dramatically if the explicit form of nonlinear functions $F_j(\vec{x},\vec{\mu}^0)$ is known and only the parameters have to be found. In this case the problem of reconstruction is set forth and may be solved with the required accuracy. This may lead to interesting applications of reconstruction problem. In particular, we further show that the method for reconstruction may be used successfully for confidential communication.

Recently, the methods for secure communication based on exploiting broadband chaotic oscillations of some dynamical chaos oscillator as a carrying (or masking) signal were suggested [4]. To extract information signal from the chaotic background the phenomenon of chaotic synchronization [5] is usually used. Another possibility to protect transmitted information may be realized by means of controlling chaos [6].

An important problem of simultaneous transfer of several information signals in a single carrier has been studied in [7]. For this purpose several parameters of DS were modulated by information signals and then extracted by means of *autosynchronization* phenomena [7].

In the present report we suggest an alternative method for demodulation of encoded messages using the technique of reconstruction of DSs from a one-dimensional realization. The idea of the method is rather simple. Let a dynamical chaotic oscillator, whose mathematical model is known [i.e., the dynamics of the oscillator is described by the known nonlinear system (1) with N phase variables and M constant parameters] be a transmitter. To transmit M messages simultaneously let us perform modulation of parameters $\mu_i^*=\mu_i^0+\mu_i(t)$, where μ_i^0, $i=1,\ldots,M$, are the constant values of control parameters of oscillator (1) and $\mu_i(t)$ are information signals performing a relatively slow parameter modulation. The signal transmitted via the communication channel is now described by the system

$$\frac{d\vec{x}}{dt}=\vec{F}(\vec{x},\vec{\mu}^0+\vec{\mu}(t)). \tag{2}$$

Let the recipient know for sure the form of nonlinear vector-function $\vec{F}(\vec{x},\vec{\mu}^0)$. In other words, the recipient of information knows the mathematical model of oscillator (1) used for information transmission. The concrete values of the parameter vector $\vec{\mu}^0$ may be unknown. The recipient of the information is able to uniquely solve the problem of reconstruction of system (2) from one-dimensional realization, which can be measured at the input of the receiver. The solution of the reconstruction problem means the extraction of modulation signals $\mu_i(t)$.

In order to get the unique reconstruction of $\mu_i(t)$, system (1) should be transformed into the form

$$\frac{dx_1}{dt}=x_2,\quad \frac{dx_2}{dt}=x_3,\quad \ldots,\quad \frac{dx_n}{dt}=f(\vec{x},\vec{\mu}^0) \tag{3}$$

by means of variable substitution. If transformations from system (1) to system (3) do not exist, then one needs to know the temporal dependences of more than one phase variable. It is indeed possible to realize this transformation for many well-known models such as Lorenz and Rössler systems and Chua's circuit (see, e.g., [2]). Here we suppose that system (1) permits transformation into the form (3).

In radiophysical tasks of information transmission the modulating signals are low frequency in comparison to the carrier. Using this circumstance, we study relatively slow parameter modulation. Let us introduce a small time interval t_0 during which the values of $\mu_i^* = \mu_i^0 + \mu_i(t)$ practically do not change. At the time scale of order t_0 the nonautonomous property of Eq. (2) may not be taken into account. As shown below, the reconstruction technique works successfully for relatively small values of t_0, i.e., for very short time series, the latter giving an opportunity to define the current parameter values μ_i^* for each t_0, i.e., to restore the modulation signals.

Consider the concrete examples. We use the Anishchenko-Astakhov oscillator as a source of chaotic oscillations [8]:

$$\frac{dx}{dt} = mx + y - xz, \quad \frac{dy}{dt} = -x, \quad \frac{dz}{dt} = -gz + 0.5g(x + |x|)x. \tag{4}$$

Bearing in mind that the signal from the oscillator being received by the recipient is the one-dimensional realization $y(t)$, we transform Eq. (4) by means of variable substitutions $Y = y$, $Z = -x$, and $X = -mx - y + xz$ into the form

$$\frac{dY}{dt} = Z, \quad \frac{dZ}{dt} = X, \quad \frac{dX}{dt} = f(X,Y,Z), \tag{5}$$

$$f(X,Y,Z) = \frac{X(X+Y)}{Z} + (mg-1)Z - g(X+Y) + 0.5g(|Z| - Z)Z^2. \tag{6}$$

The recipient of the information may define the values of $Z, X, dX/dt$ in system (5) at discrete time moments $i\Delta t$ employing numerical differentiation of the time series $y(i\Delta t) = Y(i\Delta t)$, where Δt is sampling step, $i = 1,\ldots,K$; $K = [t_0/\Delta t]$, $[t_0/\Delta t]$ being the integer part of the quantity $t_0/\Delta t$. Therefore, the problem of defining the current values of m and g is reduced to the solution of K algebraic equations (6) with two unknown quantities m, g by means of a least-squares algorithm.

Consider the simplest case. Let the value of m be fixed ($m = 1.5$) and the function f contain only one unknown quantity g. We choose the stepwise temporal dependence obtained by the scanning of Einstein's portrait [Fig. 1(a)] as a law of modulation. The range of parameter variation ($g \in [0.15, 0.25]$) was split into 256 subranges, each corresponding to the shade of the black-and-white image. To test the used method of demodulation for its robustness we added a normally distributed random value with variance 10^{-4} to $g(t)$. Figures 1(b) and 1(c) show the signal in the communication channel and the reconstructed information signal, respectively.

To prove the possibility of simultaneous transmission of several messages under the noise influence, two parameters were modulated simultaneously as follows: m was modulated by a chaotic signal from the Rössler system, where the time renormalization was performed to provide a slower process of parameter variation compared to oscillations in Eq. (4); g was modulated by a harmonic signal. The results obtained are presented in Fig. 2 and testify to the reliability of the method in the case of simultaneous transmission of two independent messages in the presence of noise.

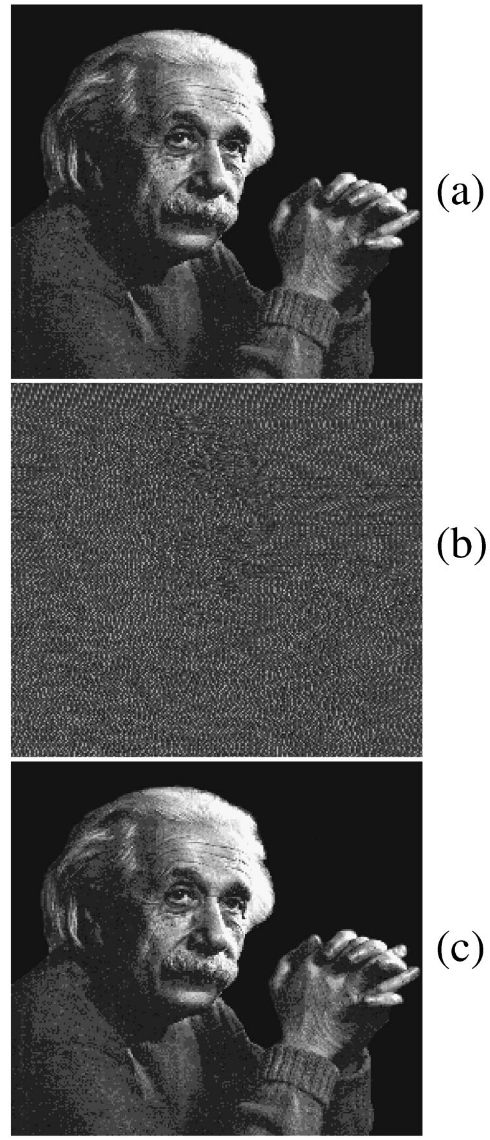

FIG. 1. (a) Initial portrait scanned with resolution 500×464, (b) the signal from the chaotic oscillator under modulation, and (c) restored portrait obtained by global reconstruction technique.

The following conclusions can be derived from the results presented. First, the proposed technique does not suppose theoretical limitations to the number of parameters being modulated simultaneously, i.e., it allows one to use the regime of simultaneous transmission of M information signals. However, a small value of sampling step Δt is necessary for the computation of parameter values with good enough accuracy (we used $\Delta t = 0.025$) as well as a rather high precision of conversion of the carrier into digital form by an analog-to-digital converter (ADC). (Of course, the higher the ADC resolution, the smaller the error of the parameter value computation.) Second, if while making transformations from Eq. (1) to Eq. (3) one needs to compute the derivatives of the parameters, the requirement of a slow modulation becomes essential [as in the case of $m(t)$ in Eq. (4)]. In this case the condition of a slow parameter modulation allows one to neglect dm/dt in comparison to dx/dt. However, for the parameter $g(t)$ the requirement of a slow modulation is not so essential. When m is fixed Eq. (6) is linear with respect to g.

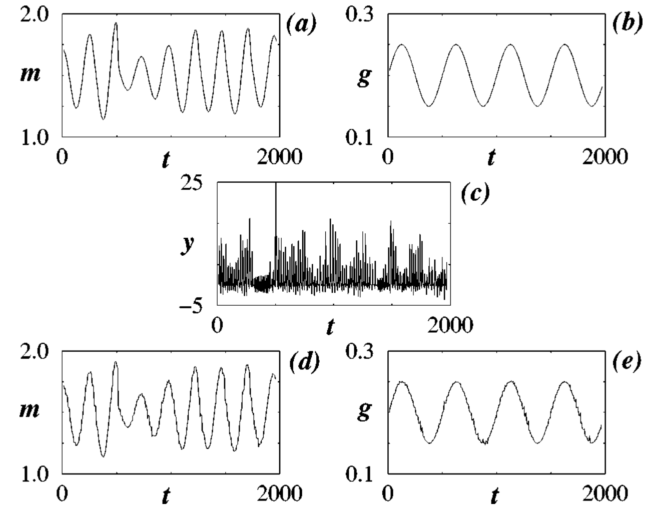

FIG. 2. (a) and (b) Information signals performing the modulation of the parameters m and g, respectively, (c) signal in the channel of communication, and (d) and (e) restored signals. To test the method from the viewpoint of its stability with respect to noise we added to both signals (a) and (b) noise with variance $D=10^{-4}$. We also added noise $D=10^{-5}$ into the first equation of system (4). t is measured in seconds.

In this case the value of g may be estimated using the knowledge of phase variables and their derivatives only at a single moment in time. Since the solution of such an equation does not require much computation, one may realize demodulation of the information signal in real time in fact.

To realize the proposed method for signal demodulation in practice, the recipient of the information must possess a specialized processor or computer and an ADC. We suppose that such a method for confidential communication may find its effective application in the exchange of information on short enough distances using a cable network, so that noise would not be created while broadcasting.

In the present paper we did not compare the described method for demodulation with the ones suggested in [7], which may be a topic of separate research. We suppose that in some situations the technique for global reconstruction offers the possibility of using a larger frequency range for modulation signals than the method of autosynchronization.

The results obtained present an alternative application of the method for dynamical model reconstruction. In this paper the method for confidential communication is illustrated for the modified oscillator with inertial nonlinearity as an example. Similar results were obtained for the Lorenz and Rössler systems, which is why we may state with confidence that the workability of the method does not depend on the choice of the source of chaotic oscillations, whose mathematical model can be transformed to the form (3).

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